Computational helioseismology

How to get our hands on the interior of the Sun

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MPS

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MATHEMATICS OF EXPERIMENT

C04: Correlations of solar oscillations: modeling and inversions

CRC 1456 Opening Symposium, March 17, 2022

Structure of the talk

Helioseismological data

- Observed data
- Preprocessed data

The forward solver

Input: excitations/parameters in the Sun
Task: compute velocity (on the surface)

- Interior PDE model and discretization
- Boundary conditions for the Sun

The inverse problem

Input: measurements on the surface
Task: compute excitation/parmeters in the Sun

- Iterative holography (advertisement)

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More details?
$\rightsquigarrow$ C04 poster presentation
(after plenary)

Helioseismological data

What is helioseismology?

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Oxford dictionary

"The study of the Sun's interior by the observation and analysis of oscillations at its surface."

global helioseismology

- input: frequencies of resonances (eigenmodes)
- yields radially symmetric (reference) model of the Sun

local helioseismology

- input: travel times / correlations of oscillations
- inverse boundary value problems, typically in the frequency domain
- 2D/3D imaging of Sun (farside/interior)

Aims:

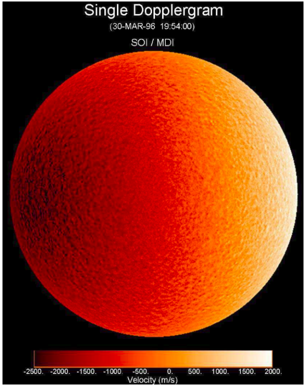
- Space weather prediction
- Understanding the solar cycle

Observed data

Dopplergrams from decades

Images of the line-of-sight velocities at the solar surface computed from Doppler shifts of certain (atomic) spectral lines.

Location	Instrument	Time	Spatial resol.	temporal resol.
SDO satellite	HMI: Helioseismic & Magnetic Imager	05/2010 – now	4096^2	45s
SOHO satellite	MDI: Michelson Doppler Imager	05/1996 – 04/2011	1024^2	60s
GONG ground based	Fourier Tachometer	1995 – now	839^2	60s

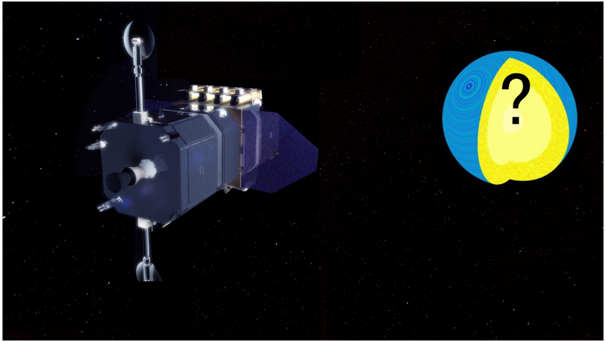


MDI image (from Gough[1])

[1] D. Gough, Vainu Bappu Memorial Lecture: What is a sunspot? 2010

Origin of oscillations

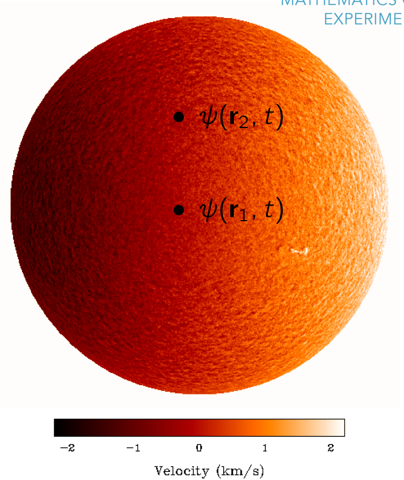
- The outer 28% of the solar interior form the solar convection zone.
- Turbulence effectively acts as random perturbation.
- Waves propagate from here to the surface where they can be observed.



Preprocessed data

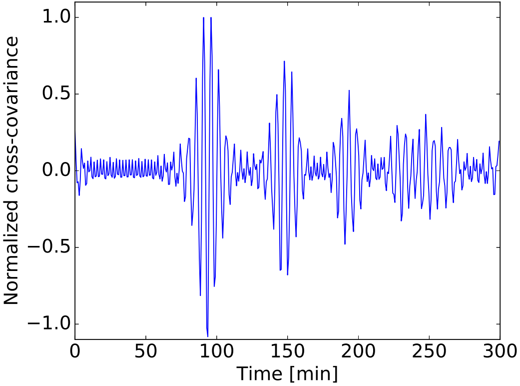
Correlations

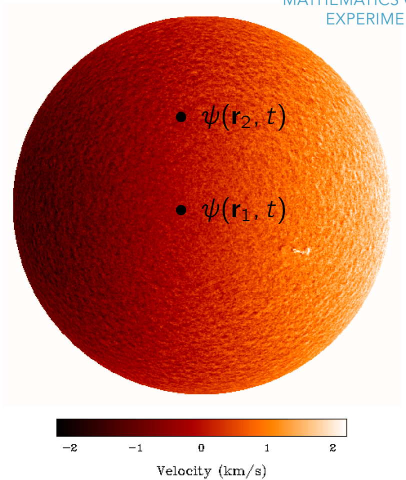
- Consider line-of-sight velocities $\psi(\mathbf{r}_i, t)$ $i = 1, 2$.



Correlations

- Consider line-of-sight velocities $\psi(\mathbf{r}_i, t)$ $i = 1, 2$.
- Cross covariance between a point $\mathbf{r}_1$ at the equator and $\mathbf{r}_2$ at a latitude of 40°.





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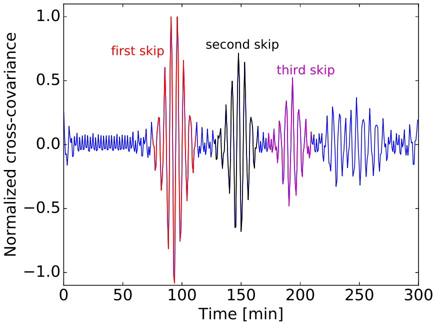
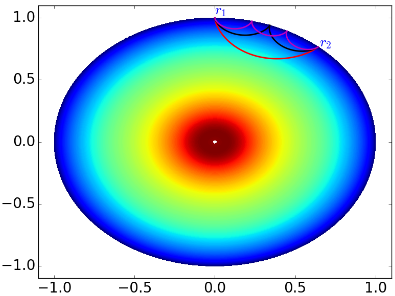




- Waves are refracted due to rapid sound speed increase

Correlation data

Cross-correlations in frequency domain

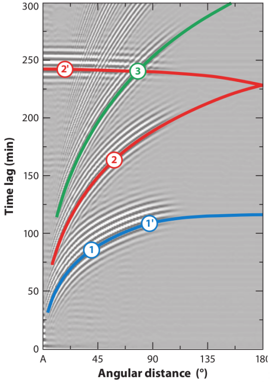
- $\psi(\mathbf{r}_1, t) \to \hat{\psi}(\mathbf{r}_i, \omega)$
- Observation time (21 years, cadence of 1 min)
  $\leadsto$ large number of available frequencies
- Cross-correlations

$$C(\mathbf{r}_1, \mathbf{r}_2, \omega) = \hat{\psi}(\mathbf{r}_1, \omega)^* \hat{\psi}(\mathbf{r}_2, \omega)$$

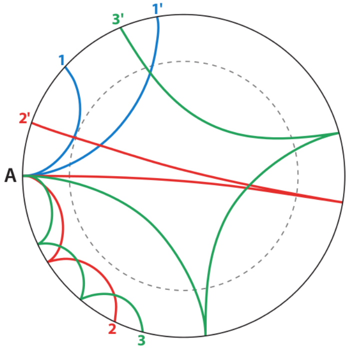
- $\mathbf{r}_1$ and $\mathbf{r}_2$ are any points of $4096 \times 4096 \Rightarrow \approx 10^{13}$ cross-correlations per frequency
- Correlation data is extremely noisy $\Rightarrow$ averaging in spatial and frequency domain

Cross-covariance in time-distance diagram:

time-distance diagram[2]



ray approximation



Time-distance helioseismology[3]:

Reduce high-dimensional correlation data to smaller number of travel-times.

[2] L. Gizon, A. C. Birch, H. C. Spruit. Local helioseismology: Three-dimensional imaging of the solar interior. ARAA, 2010.

[3] T. Duvall Jr., S. Jefferies, J. Harvey, M. Pomerantz. Time-distance helioseismology. Nature, 1993.

The forward solver:
Interior PDE model and discretization

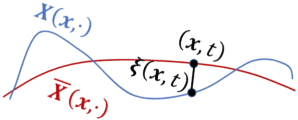
Equations of solar oscillations

Perturbed flow equations

- Assume there is a smooth stationary equilibrium $(\overline{\cdot})$ to the equations of conservation of momentum and mass and gravitational potential

  (no magnetic field yet)

- Trajectory of a particle at $\mathbf{x}$ after perturbation: $X(\mathbf{x}, t) = \overline{X}(\mathbf{x}, t) + \boldsymbol{\xi}(\mathbf{x}, t)$



- Consider Lagrangian perturbation of conservation of momentum and mass
  $R(\mathbf{x} + \boldsymbol{\xi}(\mathbf{x}, t), t) - \overline{R}(\mathbf{x}, t)$
- Eulerian perturbation of equations for gravitational potential
  $R(\mathbf{x}, t) - \overline{R}(\mathbf{x}, t)$
- and time-harmonic ansatz $\boldsymbol{\xi}(\mathbf{x}, t) = Re(\boldsymbol{\xi} \exp(-i\omega t))$

[4] D. Lynden-Bell, J. Ostriker. On the stability of differentially rotating bodies. Monthly Not. Roy. Astr. Soc., 1967

Equations of solar oscillations

We arrive at the following "monster": (similar to Galbrun's equation[5] in Aeroacoustics)

$$\rho\underbrace{(-i\omega + \mathbf{u}\cdot\nabla + \Omega\times)^2}_{=:\mathcal{D}}\boldsymbol{\xi} - \nabla(\rho c_s^2 \nabla\cdot\boldsymbol{\xi}) + (\nabla\cdot\boldsymbol{\xi})\nabla p - \nabla(\nabla p\cdot\boldsymbol{\xi})$$

$$+ (\mathrm{Hess}(p)\boldsymbol{\xi} - \rho\,\mathrm{Hess}(\phi))\boldsymbol{\xi} - i\gamma\rho\omega\boldsymbol{\xi} + \rho\nabla\varphi = \mathbf{s} \quad \text{in} \quad D,$$

$$-\frac{1}{4\pi G}\Delta\varphi + \nabla\cdot(\rho\boldsymbol{\xi}) = 0 \quad \text{in} \quad \mathbb{R}^3.$$

$\boldsymbol{\xi}$: Lagrangian displacement: difference between perturbed/unperturbed position,

φ: perturbation of gravitational potential,

c_s: sound speed,

$\mathbf{u}$: background velocity,

ϕ: gravitational background potential,

ω: frequency,

ρ: density,

Ω: angular velocity,

γ: damping,

p: pressure

+ boundary conditions

[5] H. Galbrun. Propagation d'une onde sonore dans l'atmosphère et théorie des zones de silence. 1931

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Discretization?

[5] H. Galbrun. Propagation d'une onde sonore dans l'atmosphère et théorie des zones de silence. 1931

Helmholtz-type decomposition

- straight-forward (DG) discretizations fail[6]

[6] J. Chabassier, M. Duruflé. Solving time-harmonic Galbrun's equation with an arbitrary flow. Application to Helioseismology. 2018.
[7] M. Halla, T. Hohage. On the well-posedness of [...] and the equations of stellar oscillations. SIMA, 2021

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- Let's consider the leading order terms (for $\boldsymbol{\xi}$):

$$+\rho(-i\omega + \mathbf{u}\cdot\nabla + \Omega\times)^2\boldsymbol{\xi} - \nabla(\rho c_s^2 \nabla\cdot\boldsymbol{\xi}) - i\rho\gamma\omega\boldsymbol{\xi} + \ldots = f$$

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- crucial in the analysis[7] is a Helmholtz-type decomposition of solution space

$$\mathbb{V} = \{\boldsymbol{\xi} \in \mathbf{L}^2(D) : \ \nabla\cdot\boldsymbol{\xi} \in L^2(D), \ (\mathbf{u}\cdot\nabla)\boldsymbol{\xi} \in \mathbf{L}^2(D), \ \nu\cdot\boldsymbol{\xi} = 0 \text{ on } \partial D\} = \mathbb{X} \oplus \mathbb{Y} \oplus \mathbb{Z}$$

with

- $\mathbb{X}$: (generalized) divergence-free functions
- $\mathbb{Y}$: gradients (compactly emb. in $\mathbf{L}^2$) with $\|\nabla\cdot\mathbf{y}\|_0 + \|\mathbf{y}\|_0 \gtrsim \|\mathbf{y}\|_{\mathbb{V}}$ for all $\mathbf{y} \in \mathbb{Y}$
- $\mathbb{Z}$: a finite dimensional subspace

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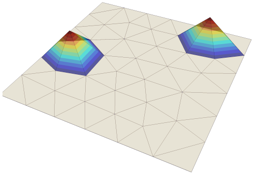
Discretizations with Helmholtz-type decomposition



Finite Element Methods

- based in calculus of variation
  $\rightarrow$ solve variational formulation over finite dimensional space
- approximate solution using polynomials on each mesh element

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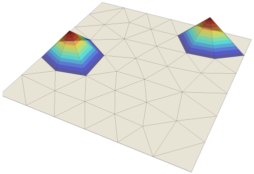

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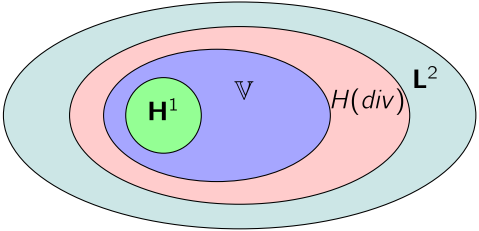
Choice of discretization (space)?

- natural choice: $\mathbb{V}_h \subset H(div)$
- $\mathbb{V}_h \subset H(div) \rightsquigarrow$ Helmholtz-type decomposition
- but: many more choices are possible...



A simple model problem

[Ma. project of T. Alemán][8]

Simplified version (real, with artificial reaction term): Find $\boldsymbol{\xi} \in \mathbb{V}$, s.t.

$$\langle c_s^2 \rho \nabla \cdot \boldsymbol{\xi}, \nabla \cdot \boldsymbol{\eta} \rangle - \underbrace{(\langle \rho(\mathbf{u} \cdot \nabla)\boldsymbol{\xi}, (\mathbf{u} \cdot \nabla)\boldsymbol{\eta} \rangle + \|\mathbf{u}\|_\infty \langle \rho \boldsymbol{\xi}, \boldsymbol{\eta} \rangle)}_{a(\boldsymbol{\xi},\boldsymbol{\eta})} = \langle f, \boldsymbol{\eta} \rangle \quad \forall \boldsymbol{\eta} \in \mathbb{V}.$$

Generic discretization ($\mathbb{V} \rightsquigarrow \mathbb{V}_h$, $\nabla\cdot \rightsquigarrow \nabla_h\cdot$, $a(\cdot,\cdot) \rightsquigarrow a_h(\cdot,\cdot)$): Find $\boldsymbol{\xi}_h \in \underline{\mathbb{V}_h}$, s.t.

$$\langle c_s^2 \rho \underline{\nabla_h} \cdot \boldsymbol{\xi}_h, \underline{\nabla_h} \cdot \boldsymbol{\eta}_h \rangle - \underline{a_h}(\boldsymbol{\xi}_h, \boldsymbol{\eta}_h) = \langle f, \boldsymbol{\eta}_h \rangle \ \forall \boldsymbol{\eta}_h \in \underline{\mathbb{V}_h}.$$

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Discrete Helmholtz decomposition

$$\mathbb{X}_h = \{\boldsymbol{\xi}_h \in \underline{\mathbb{V}_h} : \underline{\nabla_h \cdot} \boldsymbol{\xi}_h = 0\} \qquad \mathbb{Y}_h = \mathbb{X}_h^{\perp} = \{\boldsymbol{\xi}_h \in \mathbb{V}_h : \underline{a_h}(\boldsymbol{\xi}_h, \boldsymbol{\eta}_h) = 0 \quad \forall \boldsymbol{\eta}_h \in \mathbb{X}_h\}$$

$$\text{and } c\|\mathbf{y}_h\|_{\mathbb{V},h} \leq \left\|\underline{\nabla_h \cdot} \mathbf{y}_h\right\|_0 \quad \forall \mathbf{y}_h \in \mathbb{Y}_h, \qquad c \neq c(h, c_s).$$

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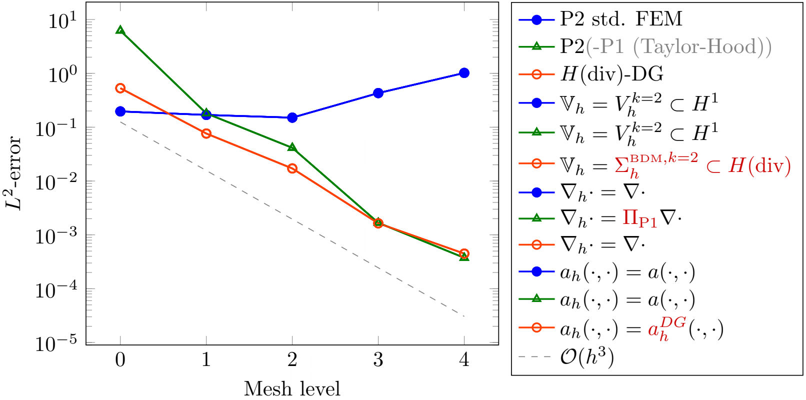
Good news: This is achieved by every inf-sup stable Stokes element!

$\rightsquigarrow$ Choose $\mathbb{V}_h$, $\nabla_h \cdot$, $a_h(\cdot,\cdot)$ suitable for a Stokes(-type) problem.

[8] T. Alemán. Robust Finite Element Discretizations for a PDE arising in Helioseismology. Master's thesis. 2022

Numerical example for simplified problem



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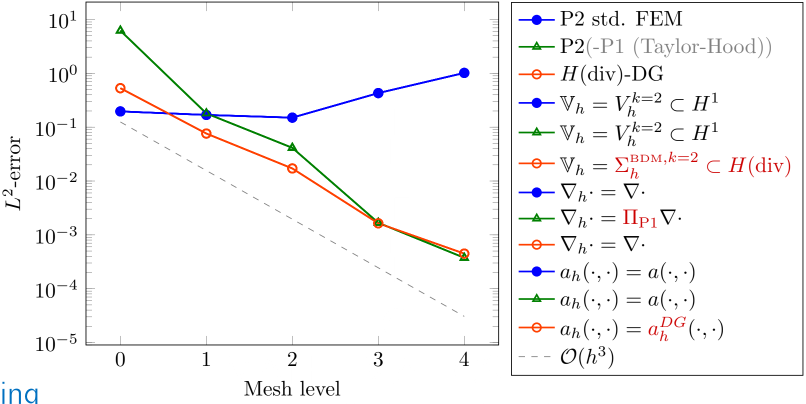

Ongoing

- Convergence analysis for full problem
- Numerics for full problem

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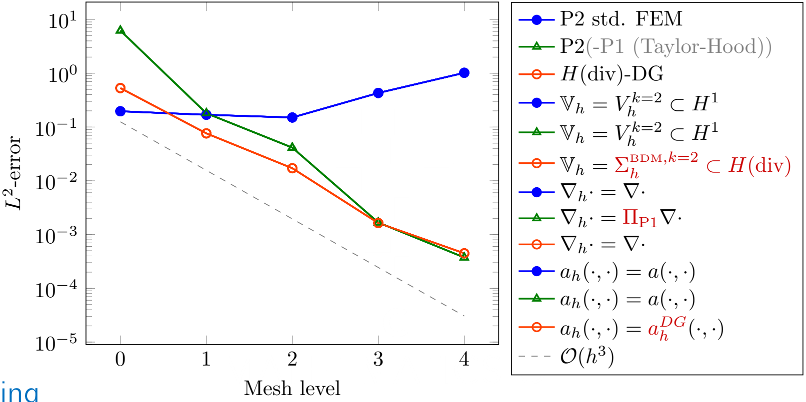

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boundary conditions?

The forward solver:
Boundary conditions for the Sun

Boundary conditions for helioseismology

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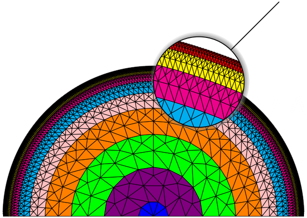
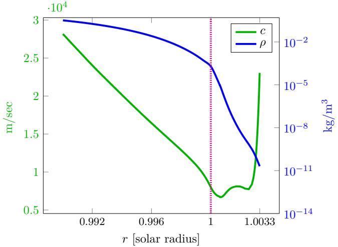
Simplified scalar model:

$u = c_s \nabla \cdot \boldsymbol{\xi}$, current working horse (no gravity effects):

$$\left(\sqrt{\rho}\Delta(\rho^{-\frac{1}{2}}) - \frac{(\omega + i\gamma)^2}{c_s^2} \right) u - \Delta u = f$$

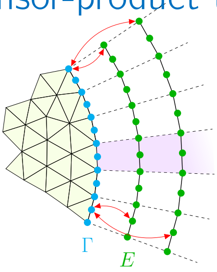
Atmosphere: No flows, spherically symmetric sound speed $c_s(r)$ and density $\rho(r)$ (separability).





Transparent bound. cond. are needed to truncate simulations close to surface!
Std. approaches (PML, loc. b.c.) fail due to rapidly changing material params..

Tensor-product type boundary conditions



- Many popular transparent b.c. follow (inherently sparse) ansatz for discretization of exterior diff. op:

$$\begin{bmatrix} A_{\Gamma\Gamma} & A_{\Gamma E} \\ A_{E\Gamma} & A_{EE} \end{bmatrix} \otimes \underbrace{M}_{\approx \mathrm{Id}_\Gamma} + \begin{bmatrix} B_{\Gamma\Gamma} & B_{\Gamma E} \\ B_{E\Gamma} & B_{EE} \end{bmatrix} \otimes \underbrace{K}_{\approx -\Delta_\Gamma}$$

- Here, $A, B \in \mathbb{C}^{(N+1)\times(N+1)}$ small matrices which encode the discrete transparent b.c.. How to choose them?

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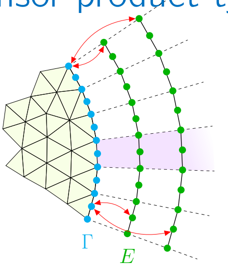

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Observation (assuming discretization in radial direction only)

- The $\mathcal{DtN}$ map (representing the exterior solution operator) can be characterized by (computable) numbers $\mathcal{dtn}(\lambda)$ $(\lambda \in \Delta_\Gamma)$
- The discrete exterior solution operator can be characterized by numbers $\mathrm{dtn}_N(\lambda) = f(\lambda, A, B)$ $(\lambda \in \Delta_\Gamma)$

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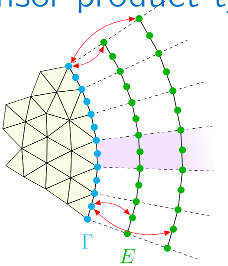

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Idea of learned infinite elements[9]:

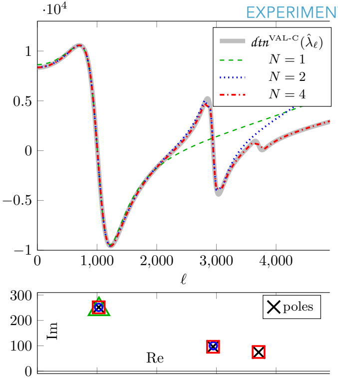
Optimize A and B to match known (computable) dtn!

Learned infinite elements [9]

- Solve rational approximation problem ($\omega_\ell \geq 0$)

$$\min_{A,B \in \mathbb{C}^{(N+1)\times(N+1)}} \sum_\ell \omega_\ell^2 |dtn(\lambda_\ell) - \mathrm{dtn}_N(\lambda_\ell)|^2.$$

- Minimizers A and B represent local element matrices of learned infinite elements.
- Error analysis guarantees[10] exponential convergence on finite intervals using only $\mathcal{O}(N)$ DOFs.



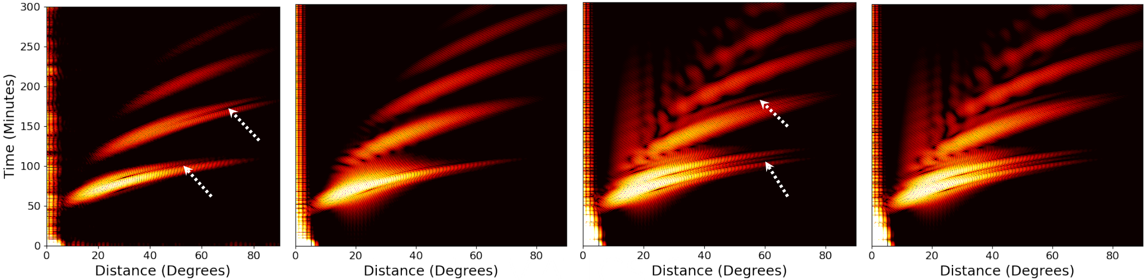
Approx. results for realistic chromospheric model

[9] T. Hohage, C.L., J. Preuß, Learned infinite elements, SISC 2021.

[10] J. Preuß, Learned infinite elements for helioseismology, PhD thesis. 2021.

Comparison

Time distance diagrams with diff. exterior domain models



(a) MDI observations

(b) `Atmo` (allows for abs. b.c.)

(c) VAL-C (meshed)

(d) Learned IE (lowest order)

[10] J. Preuß, Learned infinite elements for helioseismology, PhD thesis. 2021.

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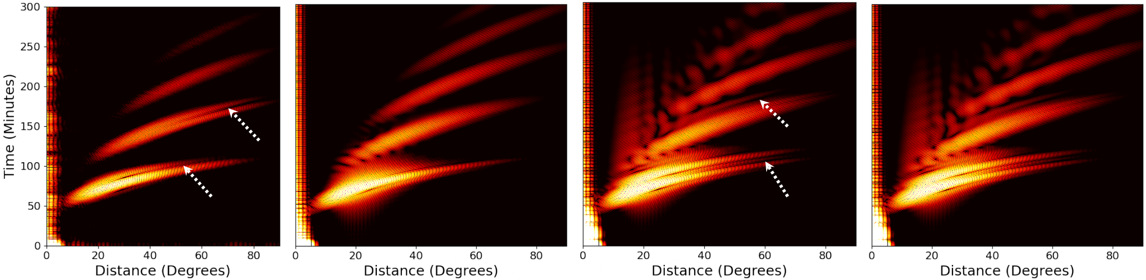

(a) MDI observations

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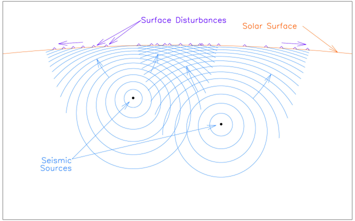
Learned infinite elements are flexible w.r.t. to atmospheric model!

10 J. Preuß, Learned infinite elements for helioseismology, PhD thesis. 2021.

The inverse problem:
Iterative holography (advertisement)

Helioseismic Holography

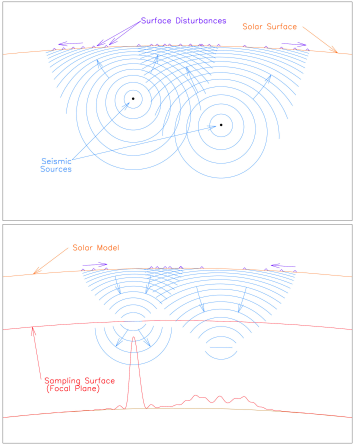


Seismic waves propagating
to/from solar surface[11].

- Forward problem: $\mathcal{L}\psi = r + s$,
  r, s: (stochastic) sources (bnd./int.), ψ wave field

[11] C. Lindsey, D. C. Braun. Seismic imaging of the sun's far hemisphere and its applications in space weather forecasting. Space Weather. 2017

Helioseismic Holography



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- Solve similar problem (backwards): $\mathcal{L}\phi = \psi$,
  where ψ takes the role of r (available data).

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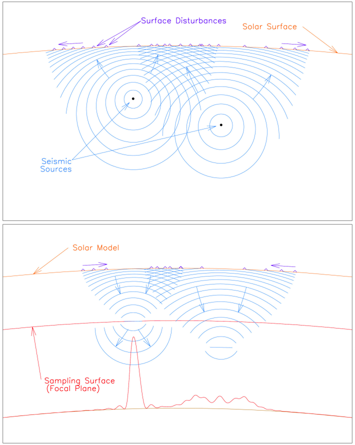

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$$\Phi_\alpha^\omega(x) = \int_A H_\alpha^\omega(x, r')\psi(r', \omega)dr',$$

H_α^ω : wave (back)propagator, A: observed part of surface

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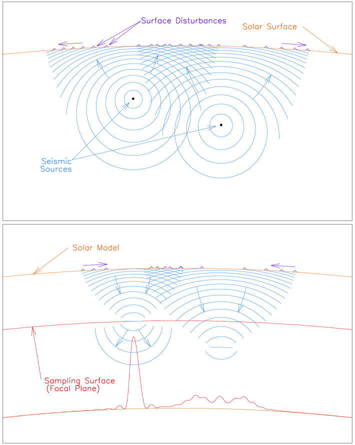

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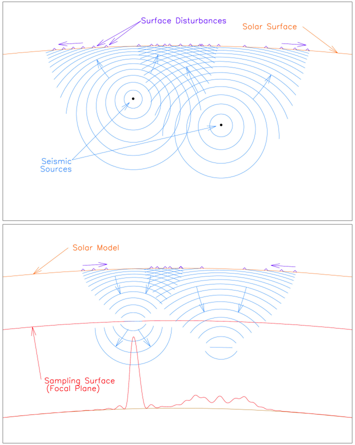

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- But: Holography is not a quantitative method

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CRC 1456 Mathematics of Experiment

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Helioseismic inverse problems

time-distance helioseismology	helioseismic holography
Dopplergrams $\hat{\psi}_i$	
↙ cross-correlation $C(\mathbf{r}_1, \mathbf{r}_2, \omega)$ ↓ travel times ↘	↘ hologram $\Phi_\alpha = H_\alpha \hat{\psi}_i$ ↓ hologram intensity
Inversion	

Helioseismic inverse problems

	time-distance helioseismology	helioseismic holography	
	Dopplergrams $\hat{\psi}_i$		
	↙	↘	
huge set of data →	cross-correlation $C(\mathbf{r}_1, \mathbf{r}_2, \omega)$	hologram $\Phi_\alpha = H_\alpha \hat{\psi}_i$	
reduction → (looses information)	↓ travel times	↓ hologram intensity	← qualitative only
	↘		
	Inversion		

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time-distance helioseismology	helioseismic holography
Dopplergrams $\hat{\psi}_i$	
↙	↘
cross-correlation $C(\mathbf{r}_1, \mathbf{r}_2, \omega)$	hologram $\Phi_\alpha = H_\alpha \hat{\psi}_i$
↓	↓
travel times	hologram intensity
↘	↙?
Inversion	

huge set of data → (cross-correlation / hologram)

reduction → (travel times / hologram intensity)
(looses information)

← qualitative only? (hologram intensity)

Helioseismic inverse problems

	time-distance helioseismology	helioseismic holography	
	Dopplergrams $\hat{\psi}_i$		
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	↘	↙?	
	Inversion		

Can we turn holography into a quantitative method?

Helioseismic inverse problems

time-distance helioseismology	helioseismic holography
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↓ travel times ↘	↓ hologram intensity ↙?
Inversion	

huge set of data → (cross-correlation)

reduction → (looses information) (travel times)

← qualitative only? (hologram intensity)

Can we turn holography into a quantitative method?

Yes! holographic backpropagation can be understood as adjoint of covariance operator

$\rightsquigarrow$ allows to applied standard iterative inverse solvers

$\Rightarrow$ quantitative helioseismic holography (by iteration)

$\Rightarrow$ Helioseismic inversion without setup of full correllation data.

$\rightsquigarrow$ details: poster session C04

Conclusion

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EXPERIMENT

Data

- Large data sets (> 20 years, cadence ≈ 1 min, 4096^2 pixel)
- Correlation data is even larger ($\rightsquigarrow$ travel times)

Forward solver

- Scalar equation is working horse (does not capture gravity)
- Learned infinite elements allow us to deal with complex atmosphere
- Vector PDE is challenging (robust discretizations & efficiency)

Inversion

- so far: time distance helioseismology and helioseismic holography
- holography made quantitative (by iteration)

Conclusion

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MATHEMATICS OF
EXPERIMENT

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Thank you for your attention!

More details? ⇝